

Abstract

Event-by-event fluctuations of nonstatistical nature provide important signals for unknown collective phenomena such as formation of quark-gluon plasma in nucleus-nucleus collisions at high energies. We propose various methods to identify nonstatistical fluctuations in the (pseudo)rapidity and the azimuthal angle distributions. The methods consist of defining appropriate measures of the fluctuations and of estimating the relevant statistical probabilities by simulation. They are applied to 3 high multiplicity nucleus-nucleus events observed by JACEE. Methods using a power spectrum or a Chebyshev expansion give fairly strong signals for nonstatistical fluctuations in all the three events Si-AgBr, Ca-C and Fe-Pb interactions. General characteristics of power spectra and Chebyshev expansion coefficients of simulated events and the effect of clustering on the fluctuations are examined in some detail.

## §1. Introduction

There are ample evidences that a large system consisting of many number of hadrons will turn into a new state of matter called quark-gluon plasma (QGP) when the energy density and/or the baryon number density becomes sufficiently high.<sup>1)</sup> High energy nucleus(A)-nucleus(A) collisions are expected to be the best process to see QGP formation in a laboratory. If this is the case, QGP may be formed in some events but not in others. Therefore it is very important to analyze the structure of the final states on event-by-event basis. The conventional single particle inclusive spectra

How to Identify Nonstatistical Fluctuations  
in High Energy Nucleus-Nucleus Collisions\*)

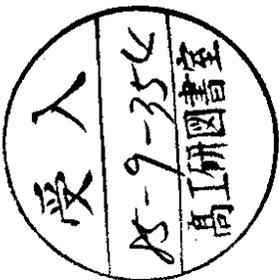
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taken over many events smear out the important information on QGP formation. The strategy is thus similar to that in jet experiments. The definition of jets and the trigger conditions are crucial there, while identification of signals on event-by-event basis is crucial in the present case. The signals proposed hitherto may be classified into three categories; (i) leptonic signals such as lepton pairs, direct photons, (ii) hadronic signals such as strange hadrons, large transverse momenta, large fluctuations, and (iii) their combinations, i.e., correlations between different signals.

However, the situation is not so simple because we have no enough knowledge about multiple production in "elementary" hadron-hadron collisions at high energies. This makes it difficult to distinguish between signals and background in some cases. In this respect, fluctuations may be most promising because they are rather free from this difficulty. Detailed knowledge on the fluctuations in hadron-hadron collisions is unnecessary. Fluctuations in A-A collisions should anyway exhibit statistical properties if the process is described by an incoherent superposition model. Any significant deviation from the statistical behavior signals the occurrence of a collective phenomenon.

Guided by these considerations, we have analyzed the fluctuations in the pseudorapidity ( $n$ ) space as well as in the azimuthal angle ( $\phi$ ) space of the high multiplicity A-A events observed by Japanese American Cooperative Emulsion Experiment (JACEE).<sup>2),3)</sup> Our method consists of defining an appropriate measure of the fluctuations, evaluating the observed value of the fluctuations by this measure and estimating the corresponding statistical probability by Monte Carlo simulation. The main results have

already been presented in previous short papers.<sup>4),5)</sup> In this paper, we present further details of our statistical analysis. Duplication with the previous papers will be avoided as much as possible. Emphasis will be put on the results using measures not used in the previous papers, the effect of the clustering and the general characteristics of the power spectra and the Chebyshev expansion coefficients of the simulated events. The characteristics of the power spectra of the "white" noise is particularly useful to estimate the relevant statistical probabilities without recourse to direct simulation.

## §2. General strategy

See Fig.1 first. It shows the pseudorapidity distributions  $dN/d\eta \equiv f(n)$  of 4 events. Actually, one of them is the real Si-AgBr

Fig. 1

event observed by JACEE<sup>2)</sup> while the others are typical events generated by simulation using a statistical model. One can hardly distinguish between them if he does not know  $f(n)$  of the real event in advance. Therefore the first impression of the reader might be that the seemingly large fluctuations of the JACEE events are merely statistical ones due to a fine binning. In fact, the fluctuations should increase as the bin width  $\Delta$  decreases. They become maximum when  $\Delta$  becomes so small that a single bin contains no particle or at most one particle in most cases. In the JACEE data,  $\Delta = 0.1$  for  $f(n)$  and  $\Delta = 10^\circ$  for the azimuthal angle distribution  $dN/d\phi \equiv g(\phi)$ ,

and most bins contain several tens of particles except for the end regions of  $f(\eta)$ . The binning is, therefore, not too fine. Thus, it requires a very careful analysis to extract a possible signal for nonstatistical fluctuations from the observed fluctuations which may be significantly masked by statistical ones.

Our strategy is rather simple. It is based on a wellknown method in statistics. Consider the pseudorapidity distribution for example. Suppose that  $f(\eta)$  of an event is given in the form of a histogram. It fluctuates from bin to bin. The values of  $f(\eta)$  at a fixed  $\eta$  fluctuate from event to event when many "equivalent" events are available. It is then reasonable to regard that  $f(\eta)$  of individual events fluctuate around a "smooth" distribution  $f_S(\eta)$ . The distribution  $f(\eta)$  is actually a set of  $L$  numbers  $\{f(\eta_i); i=1,2,3,\dots,L\}$  where  $\eta_i$  is the central coordinate of the  $i$ -th bin and  $L$  is the total number of bins. We define such a one dimensional measure  $W$  of the fluctuations in terms of  $f(\eta)$  that  $0 \leq W < \infty$  and a larger  $W$  corresponds to a larger fluctuation in some reasonable sense. When there is only one degree of freedom, i.e.,  $L=1$ , the choice  $W \equiv \{f(\eta_1) - f_S(\eta_1)\}^2$  is enough, and one may not need other definitions of  $W$ . In the present case,  $L \gg 1$  and hence there are  $L-1$  degrees of freedom even if one fixes the total multiplicity of the particles per event. Therefore, there can be many definitions of  $W$ , and one has to try them one by one. It is thus the first crucial step of our statistical analysis to define an appropriate  $W$ . The second step is to estimate by simulation the statistical probability  $P(W \geq W_{\text{Obs}})$  that  $W$  of an event is equal to or larger than the observed value  $W_{\text{Obs}}$  of the real event. In the simulation, the smooth function  $f_S(\eta)$  is used as the probability

distribution for particle production in the  $\eta$ -space. The prescription to determine  $f_S(\eta)$  is given in the next section. The corresponding smooth function  $g_S(\phi)$  for the  $\phi$ -distribution is simply supposed to be a constant.

The final step is to judge whether the observed fluctuation measured in terms of the  $W$  is a signal or not. If  $P(W \geq W_{\text{Obs}}) \leq \epsilon$ , one concludes that there is an indication for nonstatistical fluctuations. Here,  $\epsilon$  is a small positive constant, and we will adopt the most lenient choice  $\epsilon = 0.05$  for the time being. If experimental data on sufficiently many events become available from high energy heavy ion accelerators, one may easily switch to a safer choice  $\epsilon = 0.01$  or even  $10^{-3}$ .

### §3. Monte Carlo simulation

The procedure of our Monte Carlo simulation is very simple. To generate a single event, a fixed number of particles are produced in the  $\eta$ - or the  $\phi$ -space according to the probability  $f_S(\eta)$  or  $g_S(\phi)$  by using pseudorandom numbers. The same procedure is repeated to obtain the distributions  $f(\eta)$  or  $g(\phi)$  of sufficiently many "equivalent" events. The Monte Carlo data thus obtained provide the distributions of purely statistical fluctuations to be compared with the fluctuations of the real events.

To make a physically meaningful comparison, one must be careful in choosing the smooth fit  $f_S(\eta)$  though in many cases the results are rather insensitive to the choice of  $f_S(\eta)$ . In particular, we found that the result of the power spectrum analysis and that of the Chebyshev expansion are fairly sensitive to the choice of  $f_S(\eta)$ .

$$f_S(\eta) = 184 \{ (1 - e^{-5.5-\eta}) (1 - e^{-5.5+\eta}) \}^{8.1} \quad (3.4)$$

Later on, we found that (3.4) is not necessarily appropriate to estimate  $W_{\text{obs}}$  which is defined in terms of the power spectrum or the Chebyshev expansion coefficients. The truncated Chebyshev expansion is of course a better choice for  $f_S(\eta)$ . However, fluctuation measures of other kinds such as  $S$ ,  $V$  and  $\hat{f}_r$  which will be defined in the next section are insensitive to the choice of  $f_S(\eta)$ . So we will also present results obtained using (3.4) for such safe cases.

#### §4. Various measures of fluctuations

The simplest measure  $S$  for the fluctuations of  $f(\eta)$  around

$$f_S(\eta) \text{ is defined as}$$

$$S = \sum_{i=1}^L \{ f(\eta_i) - f_S(\eta_i) \}^2 \quad (4.1)$$

Though this quantity measures the total amount of deviation of  $f(\eta)$  from  $f_S(\eta)$  for a range of  $\eta$ , it is not necessarily sensitive to a jagged pattern of fluctuations. We therefore consider another quantity  $V$  which is the total vertical length of the histogram:

$$V = \sum_{i=1}^{L+1} | f(\eta_i) - f(\eta_{i-1}) |, \quad (4.2)$$

where  $f(\eta_0) = f(\eta_{L+1}) \equiv 0$ . Note that we are not interested in the horizontal component of the total length of the histogram because it is a constant independent of the bin width  $\Delta$ . On the other hand,  $V$  is a function of  $\Delta$  for a given event and hence the notion

An inappropriate choice of  $f_S(\eta)$  can easily produce a fake signal in the power spectrum or the Chebyshev expansion coefficients of the real event. In other words, a fluctuation measure  $W$  for the real event is erroneously overestimated if  $f_S(\eta)$  is chosen inappropriately. By the way,  $f_S(\eta)$  is supposed to be a statistical probability distribution which is obtained by averaging over many equivalent events with only statistical fluctuations. Therefore, we require  $f_S(\eta)$  to satisfy the following conditions: (i) it is smooth and least oscillating; (ii) it gives a best fit to  $f(\eta)$  of the real event so that  $W$ s become minimum under the condition (i). We found that the practical solution to this problem is to expand  $f(\eta)$  into a sum of Chebyshev polynomials and to take the first several terms of the expansion truncating the higher order terms:

$$f(\eta) = \frac{1}{2} a_0 + \sum_{k=1}^{L-1} a_k T_k(z), \quad (3.1)$$

$$f_S(\eta) = \frac{1}{2} a_0 + \sum_{k=1}^m a_k T_k(z) \equiv f^{(m)}(\eta), \quad (m < L-1) \quad (3.2)$$

$$z = (2\eta - \eta_{\min} - \eta_{\max}) / (\eta_{\max} - \eta_{\min}), \quad (3.3)$$

where  $T_k(z)$  is the Chebyshev polynomial of degree  $k$  and  $f(\eta)$  is given for the range  $\eta_{\min} \leq \eta \leq \eta_{\max}$ . We thus found the smooth fits  $f^{(4)}(\eta)$ ,  $f^{(5)}(\eta)$  and  $f^{(8)}(\eta)$  for the three JACEE events, respectively. See Figs. 2 to 4.

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Figs. 2 ~ 4  
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In the first paper, we used  $f_S(\eta)$  of the following form for the Si-AgBr event:<sup>4)</sup>

Therefore,  $\phi(\omega)$  for  $0 \leq \omega \leq (2\Delta)^{-1}$  contains all the information on the fluctuations. In general,  $\phi(\omega)$  for this range of  $\omega$  has a number of peaks and they represent some aspects of the fluctuations. A simple measure of the fluctuations which can be defined in terms of  $\phi(\omega)$  is the height of the ranked peaks. Suppose that  $\phi(\omega)$  has peaks at  $\omega = \omega_1, \omega_2, \dots, \omega_K$ . If the rank of the peak at  $\omega = \omega_i$  is  $r(i)$ , one defines

$$\phi(\omega_i) \equiv \hat{\phi}_{r(i)}, \quad (4.7)$$

where

$$\hat{\phi}_1 \geq \hat{\phi}_2 \geq \hat{\phi}_3 \dots \geq \hat{\phi}_K. \quad (4.8)$$

As will be shown in the next section, peaks of  $\phi(\omega)$  distribute uniformly in the  $\omega$ -space if the fluctuations are purely statistical. Therefore, concentration of a number of prominent peaks into a restricted range of  $\omega$  can be used as a measure of the fluctuations. Concentration into a low frequency region is particularly interesting because several models for decays of QGP predict the density fluctuations of fairly long wave lengths. 6), 7) We therefore define a concentration  $C_p(s/\omega_c)$  which is the number of peaks that have ranks not greater than  $s$  and lie in the range  $0 \leq \omega \leq \omega_c$ .

The power spectrum seems to be an ideal tool when the smooth fit is a constant. The situation changes however when the smooth fit has a maximum in the central region and has diminishing tails in the end regions. In such a case, the Chebyshev expansion itself is a better choice in some sense. In fact, the coefficients of the higher degree Chebyshev polynomials,  $a_k$  for  $m+1 \leq k \leq L-1$  carry all the information on the fluctuations of  $f(\eta)$  around  $f_S(\eta) = f^{(m)}(\eta)$

of the fractal dimension may be useful to analyze the  $\Delta$ -dependence of  $V$ .

Both  $S$  and  $V$  are defined by a summation of a function of  $f(\eta)$  and  $f_S(\eta)$  over all the bins. This is, however, not the only method to define a one dimensional quantity which reflects the overall pattern of the fluctuations. Another useful method is to rank a set of quantities which contain all the information on the fluctuations in order of their magnitude. The simplest example is to rank  $L$  quantities  $f(\eta_i)$  for  $i=1, 2, \dots, L$  in order of their magnitude. That is, if the rank of  $f(\eta_i)$  is  $r(i)$ , one defines

$$f(\eta_i) \equiv \hat{f}_{r(i)}, \quad (4.3)$$

where

$$\hat{f}_1 \geq \hat{f}_2 \geq \hat{f}_3 \dots \geq \hat{f}_L \quad (4.4)$$

A high-ranked  $\hat{f}_r$ , i.e.,  $\hat{f}_r$  with a small  $r$  is especially useful as a measure of local high density fluctuations.

If the fluctuations of  $f(\eta)$  around  $f_S(\eta)$  are purely statistical, they may be regarded as a kind of a white noise. A conventional method to extract a signal from a noise is to calculate the power spectrum. It is defined in the present case as

$$\phi(\omega) = (\Delta^2/2) \left\{ \sum_{k=1}^L \exp(2\pi i \omega \eta_k) \{ f(\eta_k) - f_S(\eta_k) \} \right\}^2, \quad (4.5)$$

for  $0 \leq \omega \leq (2\Delta)^{-1}$ . By the way, it is easily seen that

$$\phi\left(\frac{1}{\Delta} - \omega\right) = \phi(\omega), \quad (4.6a)$$

$$\phi\left(\omega + \frac{\pi}{\Delta}\right) = \phi(\omega) \quad \text{for } n = 1, 2, 3, \dots \quad (4.6b)$$

given by (3.2). The advantage of the Chebyshev expansion is that the fluctuation pattern reproduced in the  $\eta$ -space from the signals manifested in the large coefficients behaves well in the end regions. 5) On the other hand, the pattern reproduced from the prominent peaks in  $\phi(\omega)$  shows occasionally an abnormal behavior in the end regions. 4) The disadvantage of the Chebyshev analysis is that the distributions of  $a_k$ s for purely statistical fluctuations are not so simple as those of  $\phi(\omega)$  as will be shown in the next section.

Useful measures are now provided by ranking the higher degree coefficients  $a_k$ s in order of magnitude of the absolute values. If the rank of  $a_k$  is  $r(k)$ , one has

$$|a_k| \equiv \hat{a}_{r(k)} \quad \text{for } m+1 \leq k \leq L-1, \quad (4.9)$$

where

$$\hat{a}_1 \geq \hat{a}_2 \geq \hat{a}_3 \geq \dots \geq a_{L-m-1}. \quad (4.10)$$

One measure is provided by the high-ranked coefficients  $\hat{a}_r$  with small  $r$  and another is the concentration  $C_T(p/k_C)$  which is the number of  $a_k$ s with  $m+1 \leq k \leq k_C$  ( $< L-1$ ) and  $r(k) \leq p$ .

#### §5. Fluctuations of the JACEE events

##### 5a. Fluctuations measured by S

The S-distribution of 1000 events generated by simulation for the Si-AgBr interaction is shown in Fig.5. It is Gaussianlike. The value of S of the real JACEE event is indicated by an arrow.

There is nothing abnormal. Similarly, both Ca-C and Fe-Pb events

##### Fig. 5

exhibit no signals as long as S is concerned.

##### 5b. Fluctuations measured by V

The V-distribution of the simulated events is also Gaussianlike. There is no signals for nonstatistical fluctuations. The result for the Si-AgBr case is shown in Fig.6. The  $\Delta$ -dependence of  $V = V(\Delta)$  is shown in Fig.7. From this result, one may calculate the fractal

##### Fig. 6 and 7

dimension D by the formula:

$$D = 1 - \frac{\Delta}{V} \frac{dV}{d\Delta}. \quad (5.1)$$

Comparison with simulation has not been done yet.

##### 5c. Fluctuations measured by $\hat{f}_r$

The pseudorapidity distribution of the JACEE Si-AgBr event has a prominent peak at  $\eta \approx 0.9$ . This corresponds to large values of  $\hat{f}_1$  and  $\hat{f}_2$ . The result of simulation with  $f_S(\eta) = f^{(4)}(\eta)$  for the  $\hat{f}_r$ -distribution is shown in Fig.8. Among 1000 simulated events,

##### Fig. 8

there are only 43 events of which  $\hat{f}_2$  are equal to or greater than the observed value. One may therefore conclude that this is a signal. Other events do not show any strong signal as long as the  $\hat{f}_r$ -distribution is concerned.

##### 5d. Fluctuations measured by power spectra

As an example, the power spectrum  $\phi(\omega)$  of  $g(\phi)$  of the JACEE

Fe-Pb event is shown in Fig.9. It was first calculated and analyzed by Miyamura.<sup>3)</sup> He suggested that 4 prominent peaks in  $\phi(\omega)$  may imply a nontrivial structure in  $g(\phi)$  which is shown by the oscillatory curve in Fig.10. According to our method, the

Figs. 9 and 10

largest fluctuations in  $\phi(\omega)$  are characterized by

$$\hat{\phi}_4 = 6.12, \quad (5.2)$$

and

$$C_p(4/0.0234) = 4. \quad (5.3)$$

The simulation gives

$$P(\hat{\phi}_4 \geq 6) = 5.1 \pm 0.7 \%, \quad (5.4)$$

$$P(C_p(4/0.0234) = 4) = 1.8 \pm 0.42 \%. \quad (5.5)$$

Here, it may be useful to examine some general properties of  $\phi(\omega)$  of the simulated events. First of all, the mean power spectrum  $\langle \phi(\omega) \rangle$  and the dispersion  $D(\omega) \equiv \langle \phi^2(\omega) \rangle - \langle \phi(\omega) \rangle^2$  are essentially independent of  $\omega$  except for  $\omega = 0$ . This implies that the noises generated by simulation are indeed white. See Fig.9. The deviation at  $\omega = 0$  is merely due to the normalization condition

$$\int_0^{360^\circ} d\phi \{g(\phi) - g_S(\phi)\} = 0.$$

The distribution of the values of  $\phi(\omega)$  at any  $\omega$  is a beautiful exponential  $\langle \phi \rangle^{-1} \exp(-\phi/\langle \phi \rangle)$  as is shown in Fig.11 for the Si-AgBr case. The corresponding distribution for the real event is also shown there.

Fig. 11

Another interesting characteristic of the power spectrum is the number  $n_p$  of peaks and its distribution. The  $\phi(\omega)$  of  $g(\phi)$  of the JACEE Fe-Pb event has 11 peaks for  $0 \leq \omega \leq 0.05$ . According to the simulation, the  $n_p$ -distribution is consistent with a Gaussian (see Fig.12) and the mean number of peaks per event is  $\langle n_p \rangle = 11.58 \pm 0.11$ . From this result, one can easily evaluate the

Fig. 12

approximate values of the statistical probabilities for the concentration  $C_p(s/\omega_c)$  as follows.

For example, let estimate  $P(C_p(4/0.0234) = k)$  for  $0 \leq k \leq 4$ . Suppose that 11.58  $\sim$  12 peaks of any ranks distribute uniformly in the range  $0 \leq \omega \leq 0.05$ . On the average,  $(0.0234/0.05) \times 12 \approx 5.6$  peaks will lie in the range  $0 \leq \omega \leq 0.0234$ . Therefore, 5 or 6 peaks will lie in this range with the probabilities 0.4 or 0.6, respectively, if one disregards the more fluctuating cases. Call a peak with the rank  $r \leq 4$  a high-ranked one. Consider how many high-ranked peaks will be contained in the lower  $\omega$  region. There are of course 4 high-ranked peaks as a whole. The number of cases where  $k$  high-ranked peaks are contained among the  $l$  peaks that lie in the lower  $\omega$  region and the remaining  $4-k$  high-ranked peaks are found among  $12-l$  peaks that lie in the higher  $\omega$  region is  ${}^l C_k \times {}^{12-l} C_{4-k}$ . The total number of the unconditional cases where the 4 high-ranked peaks distribute among 12 peaks in every possible ways is  ${}^{12} C_4$ . One thus has

$$P(C_p(4/0.0234) = k) = (0.4 \times {}^5 C_k \times {}^7 C_{4-k} + 0.6 \times {}^6 C_k \times {}^6 C_{4-k}) / {}^{12} C_4. \quad (5.6)$$

The numerical result is shown and compared with the result from the simulation in Table 1. The agreement is very good indicating

Table 1

the validity of the suppositions used in deriving (5.6).

The mean peak numbers  $\langle n_p \rangle$  of  $\phi(\omega)$  of the Monte Carlo generated  $f(\eta)$  for the three interactions Si-AgBr, Ca-C and Fe-Pb are  $23.14 \pm 0.22$ ,  $33.55 \pm 0.26$  and  $14.45 \pm 0.17$ , respectively. The  $n_p$ -distributions are again consistent with Gaussian. On the other hand, the  $\phi(\omega)$  of the real events have 20, 32 and 11 peaks, respectively. There is nothing anomalous as long as  $n_p$  is concerned.

5e. fluctuations measured by Chebyshev expansion coefficients

As an example, the Chebyshev expansion coefficients  $a_k$  for  $m+1 \leq k \leq L-1$  of the JACEE Si-AgBr event are shown in Fig.13. They

Fig. 13

distribute almost at random. However, a clear tendency emerges if one pays attention to those  $a_k$ s with large absolute magnitude, i.e., the high-ranked  $\hat{a}_r$ s. The  $a_k$ s with  $r(k) \leq 13$  are shown by solid circles in Fig.13. They apparently concentrate into the low  $k$  region. In this case, however,  $a_k$ s of the simulated events have the same tendency. The distribution of  $a_k$  of the simulated events shows a Gaussianlike behavior for any  $k$  (see Fig.14) but the width of the distribution depends on  $k$  i.e., the dispersion  $\sqrt{D_k} \equiv \langle a_k^2 \rangle - \langle a_k \rangle^2$  decreases as  $k$  increases. See Fig.15. Therefore, only

Figs. 14 and 15

a careful simulation can determine whether the observed concentration

is a signal or not. We found that the strongest concentration in the Si-AgBr case is

$$C_T(13/31) = 9, \quad (5.7)$$

while the simulation gives

$$P(C_T(13/31) \geq 9) = 4.2 \pm 0.9 \%. \quad (5.8)$$

We have also checked that the magnitude of the 13 high-ranked coefficients  $\hat{a}_r$  for  $1 \leq r \leq 13$  is itself strongly enhanced in comparison with the corresponding  $\hat{a}_r$ s obtained from simulation. In Fig.16, the mean values and the dispersions of  $\hat{a}_r$  of the simulated events are shown in comparison with  $\hat{a}_r$ s of the real event.

Fig. 16

5f. effects from clustering

It is wellknown that, in multihadron production in hadron-hadron collisions, there is a positive two-particle correlation of short range nature. Since there is no reason to expect that such "a clustering effect" does not exist in nucleus-nucleus collisions, one has to check how the distributions of fluctuations change when the clustering effect is taken into account in the simulation.

We have considered the case where all the particles are produced in pairs, i.e., the case with the charged cluster size =2. If one particle is produced at  $\eta = \eta_1$  with the relative probability  $f_S(\eta_1)$ , its partner is produced at  $\eta = \eta_2$  with the probability proportional to  $\exp(-(\eta_1 - \eta_2)^2 / \delta^2)$ . A reasonable estimate for  $\delta$  consistent with experiment is  $\delta = 0.9 \sim 1.3$ . The

results shown below are for the choice  $\delta = 1.2$ .

Our first finding is that the clustering has essentially no effect on the fluctuations measured in terms of  $S$ ,  $V$  and  $\hat{f}_r$ . On the other hand, it has an appreciable effect on both the power spectra and the Chebyshev expansion coefficients. There is a subtle point in simulating particle production with the clustering effect. Because of the two-particle correlation with a finite correlation length, the limiting statistical distribution of the simulated events deviates inevitably from  $f_g(\eta)$ . This smearing effect is strongest at the end regions and yields a fake signal in  $\phi(\omega)$  if one still regards  $f_g(\eta)$  as the statistical limit to be subtracted from  $f(\eta)$ s of the simulated events. To eliminate this effect and to see a pure effect from the clustering on the power spectra, we enlarge the range of the  $\eta$ -space from  $[\eta_{\min}, \eta_{\max}]$  to  $[\eta_a, \eta_b]$  such that  $\eta_a < \eta_{\min} < \eta_{\max} < \eta_b$  and both  $\eta_{\min} - \eta_a$  and  $\eta_b - \eta_{\max}$  are greater than the correlation length. Then the clusters are produced at random and uniformly between the range  $[\eta_a, \eta_b]$  until the number of particles produced in the range  $[\eta_{\min}, \eta_{\max}]$  becomes the specified one. The power spectrum is calculated from  $f(\eta)$  for  $\eta_{\min} \leq \eta \leq \eta_{\max}$ . In this case, the statistical limit of  $f(\eta)$  is a constant for  $\eta_{\min} \leq \eta \leq \eta_{\max}$  so that there is no difficulty in the subtraction procedure.

The clustering effect on  $\langle \phi(\omega) \rangle$  is shown in Fig.17 for the Si-AgBr case. The clustering enhances  $\langle \phi(\omega) \rangle$  at the low  $\omega$  region by several tens percent but has essentially no effect at the higher  $\omega$  region,  $\omega \gtrsim 0.4$ . The effect on the distribution of the concentration  $C_T(p/k_C)$  is even weaker as is shown in Fig.18. From these

results, we conclude that the clustering effect does not change our conclusion on the possible signals for nonstatistical fluctuations.

#### §6. Discussions

In this paper, we have proposed various methods of statistical analysis to extract signals for nonstatistical fluctuations from a few observed events. The utility of the various measures of fluctuations may change case by case. In the case of the JACEE events, we found that the power spectra and the Chebyshev expansion are most useful in identifying possible signals. In fact, the pseudorapidity distributions of both the Si-AgBr and the Ca-C events and the azimuthal angle distribution of the Fe-Pb event exhibit fairly strong signals through these measures.

It is obvious that a very high multiplicity of particles per event is a big advantage in a statistical analysis. The measures  $S$ ,  $V$ ,  $\phi(\omega)$  and  $a_k$  should be useful only when the multiplicity and hence the average density is sufficiently high and the range of the (pseudo)rapidity is sufficiently wide. However, the measure  $\hat{f}_r$  may be rather free from such restrictions. It may even be useful in analyzing the fluctuations in hadron-hadron collisions where the multiplicity is much lower than that in nucleus-nucleus collisions at comparable energies. However, in a hadron-hadron case, the simulation must be carried out more carefully than in a nucleus-

nucleus case. This is because the determination of the smooth fit may be more difficult and the kinematical effects (e.g. energy momentum conservation) as well as the dynamical effects (e.g. the clustering) should be more important.

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Table 1. Statistical probabilities in percent that  $k$  high-ranked peaks lie in the range  $0 \leq \omega \leq 0.0234$ .

$k$	formula (5.6)	simulation (1000 events)
0	4.7	$6.5 \pm 0.8$
1	28.7	$28.5 \pm 1.7$
2	44.2	$42.6 \pm 2.1$
3	20.2	$20.6 \pm 1.4$
4	2.2	$1.8 \pm 0.4$

Figure captions

- Fig.1. Pseudorapidity distributions for one real event and three typical simulated events of the Si-AgBr interaction.
- Fig.2. Dashed line: the smooth fit  $f^{(4)}(\eta)$  for the Si-AgBr event. Solid line: the pattern of nonstatistical fluctuations indicated from the concentration analysis.<sup>5)</sup>
- Fig.3. The same as in Fig.2 for the Ca-C event with the smooth fit  $f^{(5)}(\eta)$ .
- Fig.4. The solid line is the smooth fit  $f^{(8)}(\eta)$  for the Fe-Pb event.
- Fig.5. Distribution of S of the events generated by simulation for the Si-AgBr interaction.
- Fig.6. Distribution of V of the same as in Fig.5.
- Fig.7. Dependence on the bin width of V for the Si-AgBr event.
- Fig.8. The  $\hat{f}_F$ -distribution of the simulated events for the Si-AgBr event. The open circles indicate the points which correspond to the JACEE event.
- Fig.9. Solid curve: the power spectrum  $\phi(\omega)$  of the azimuthal angle distribution of the JACEE Fe-Pb event. Dashed and dash-dotted curves:  $\langle\phi(\omega)\rangle$  and  $\langle\phi(\omega)\rangle + D(\omega)$ , respectively, of the simulated events.
- Fig.10. The azimuthal angle distribution of the Fe-Pb event. The curve shows an oscillatory structure suggested from a power spectrum analysis.<sup>3)</sup>
- Fig.11. Open circles: distribution of  $\phi(\omega)$  of the simulated events for the Si-AgBr interaction. Solid circles: the same for the real event.

- Fig.12. The distribution of the number of peaks in  $\phi(\omega)$  of the simulated events for  $g(\phi)$  of the Fe-Pb interaction.
- Fig.13. The Chebyshev expansion coefficients of higher degrees for the Si-AgBr event. Solid circles show the 13 high-ranked coefficients.
- Fig.14. The distributions of  $a_k$ s of the simulated events for the Si-AgBr interaction.
- Fig.15. The dispersion  $D_k$  of the Chebyshev expansion coefficients of the simulated Si-AgBr events.
- Fig.16. The ranked coefficients  $\hat{a}_k$  (solid circles) of the Si-AgBr event are shown in comparison with the mean  $\hat{a}_k$  (open circles) of the simulated events. The bars indicate  $\langle\hat{a}_k^2\rangle \pm (\langle\hat{a}_k^2\rangle)^{1/2}$ .
- Fig.17. Solid curve: the mean power spectrum without clustering. Dashed curve: the same with clustering.
- Fig.18. The distribution of  $C_T = C_T(13/31)$  of the simulated events. The dashed and the solid histograms are for the cases with and without clustering, respectively.

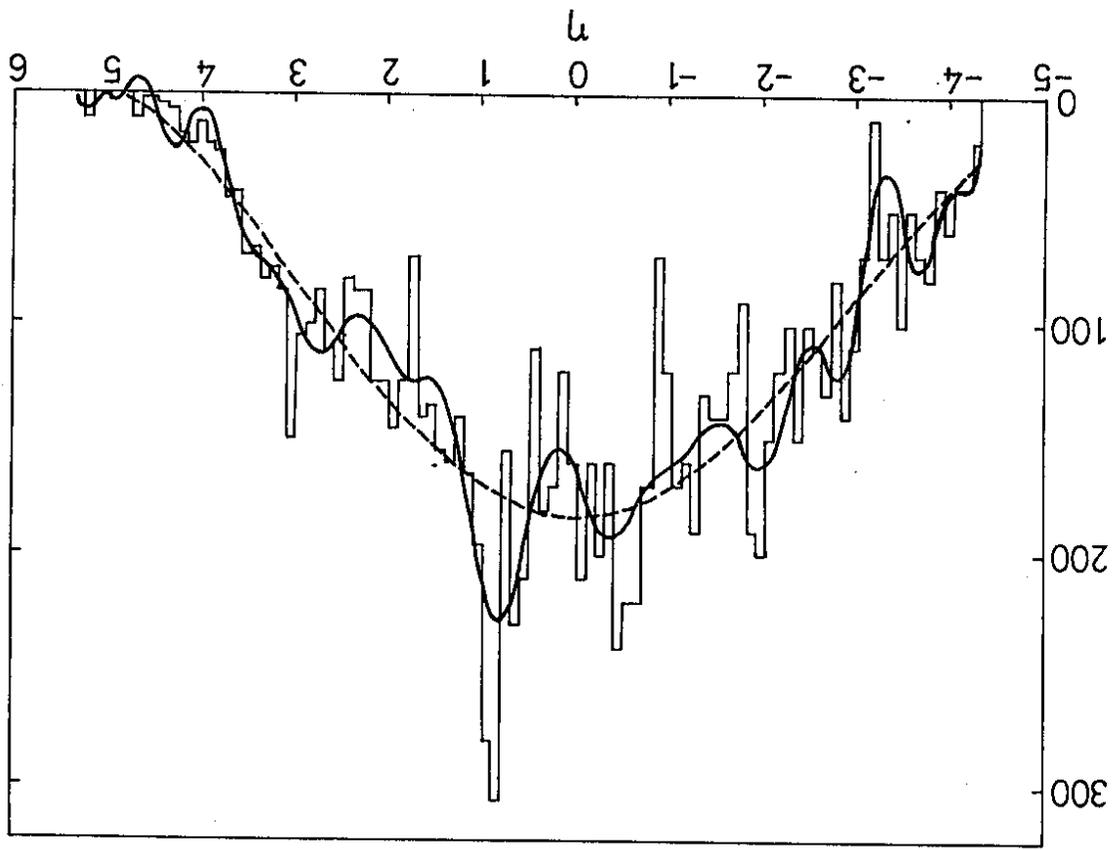


Fig. 2

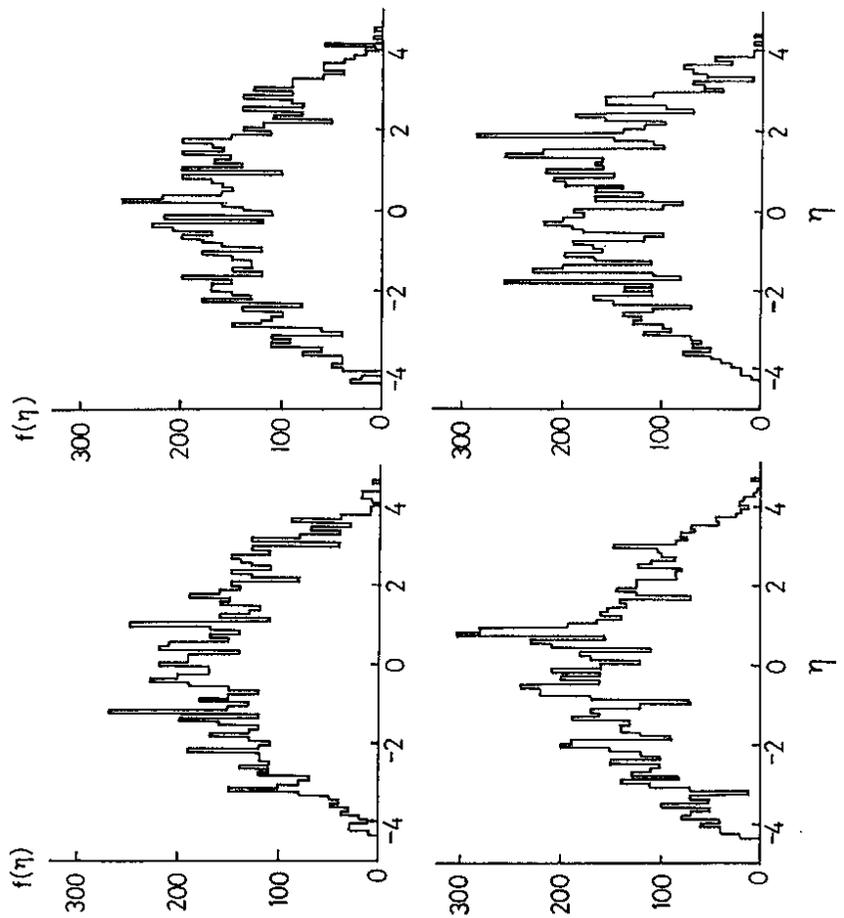


Fig. 1

Fig. 6

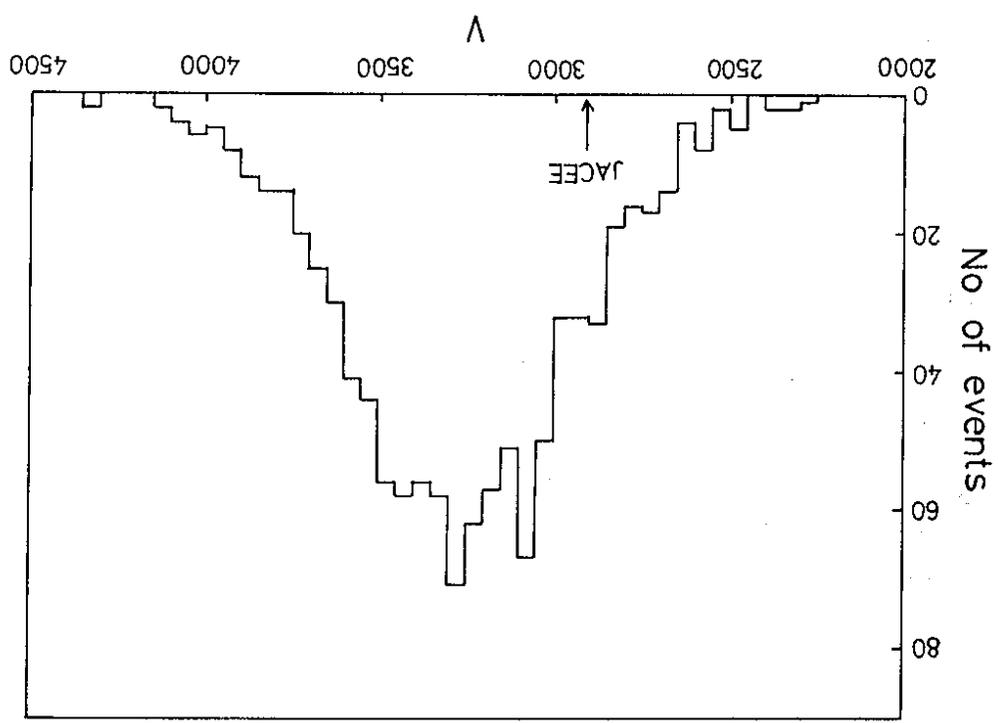
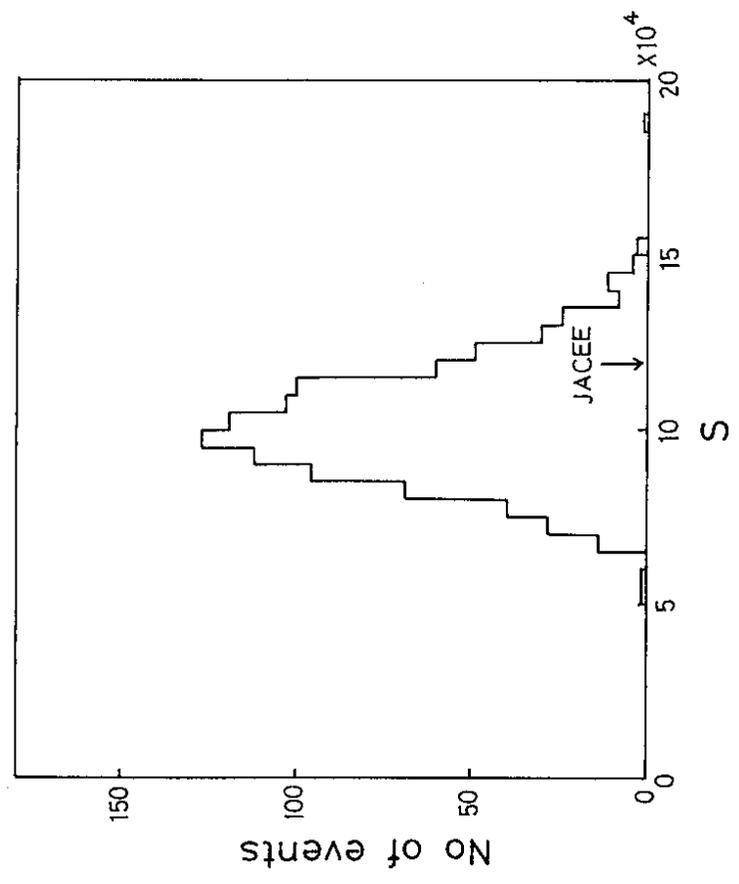


Fig. 5



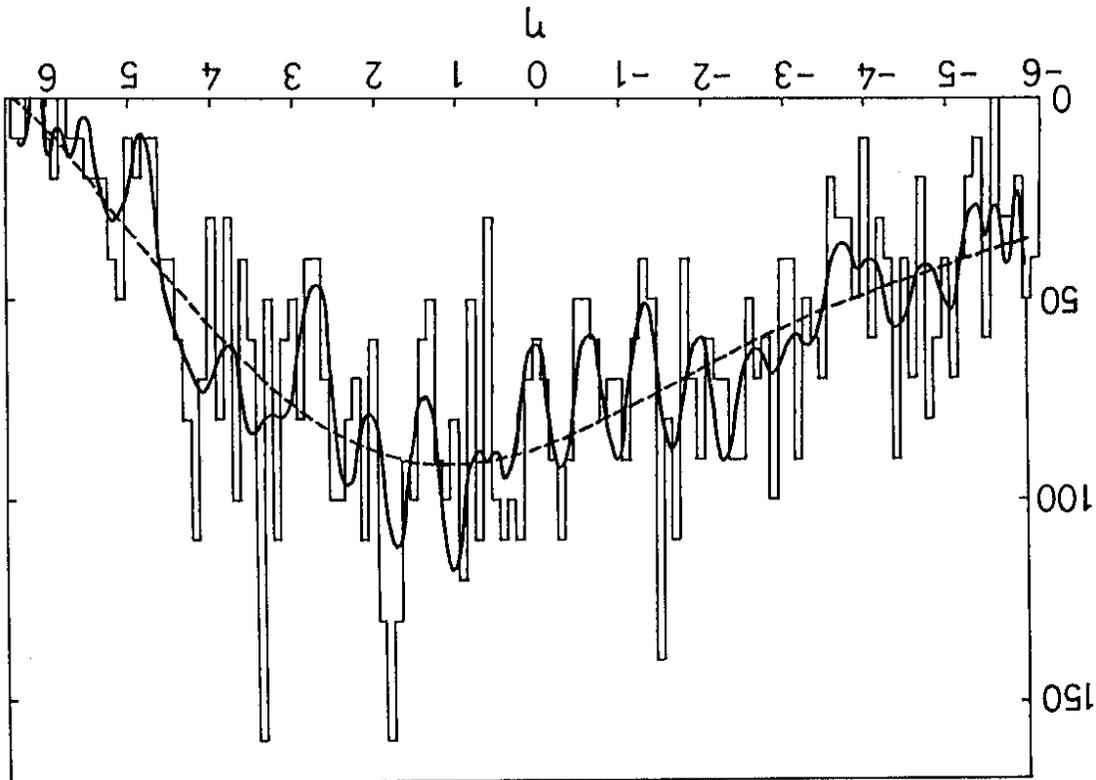


Fig. 3

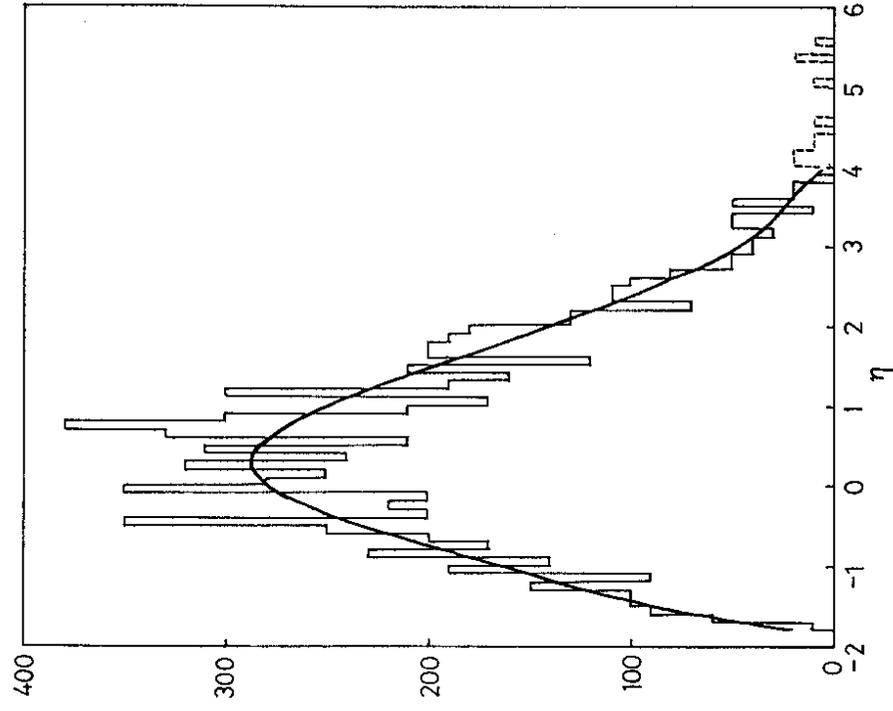


Fig. 4

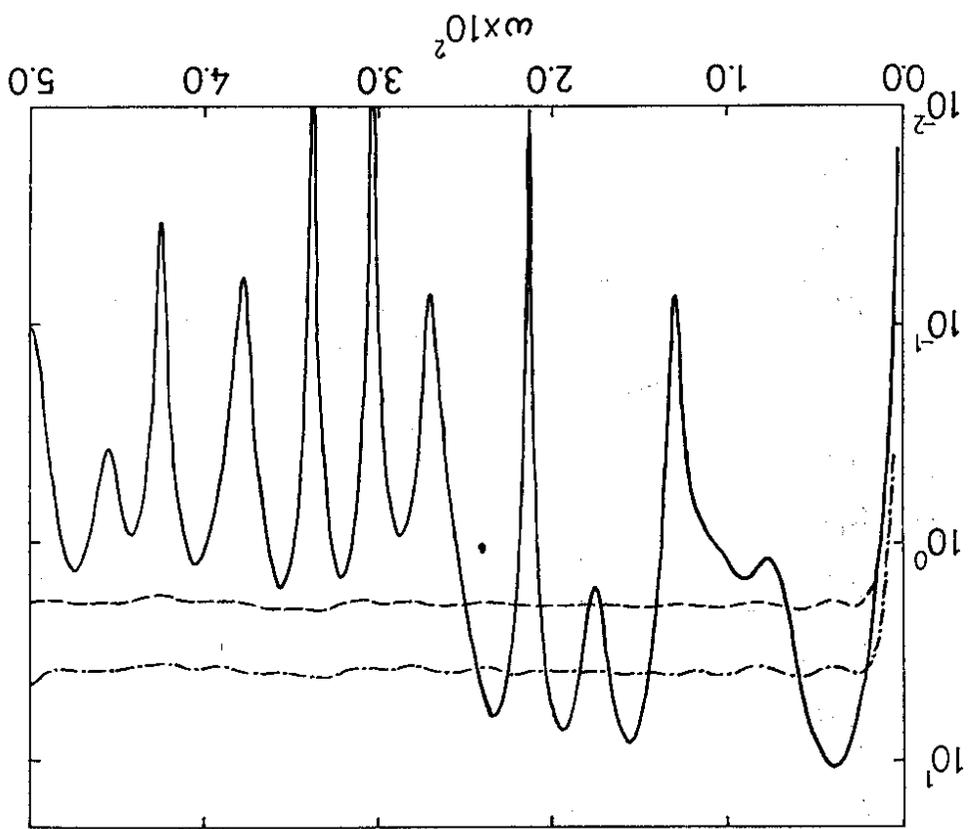


Fig. 9

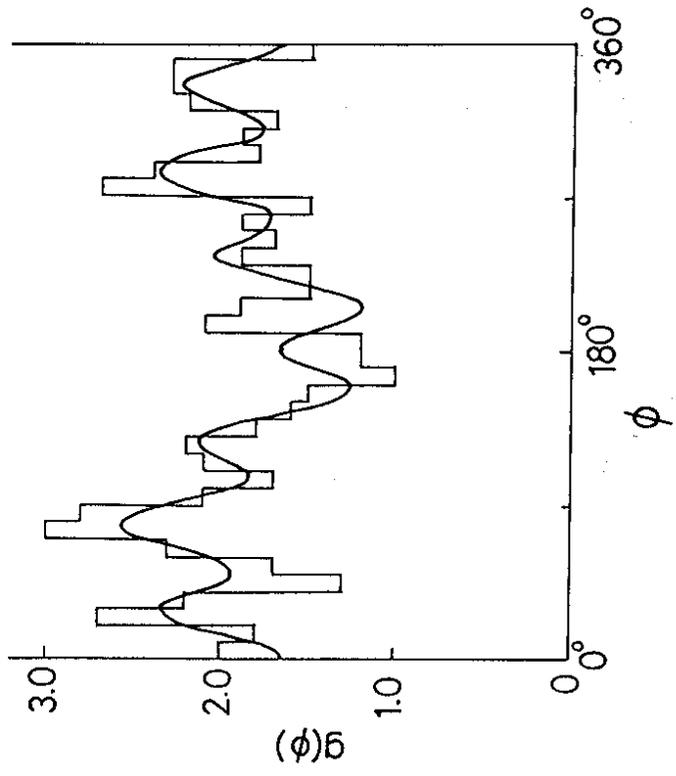


Fig. 10

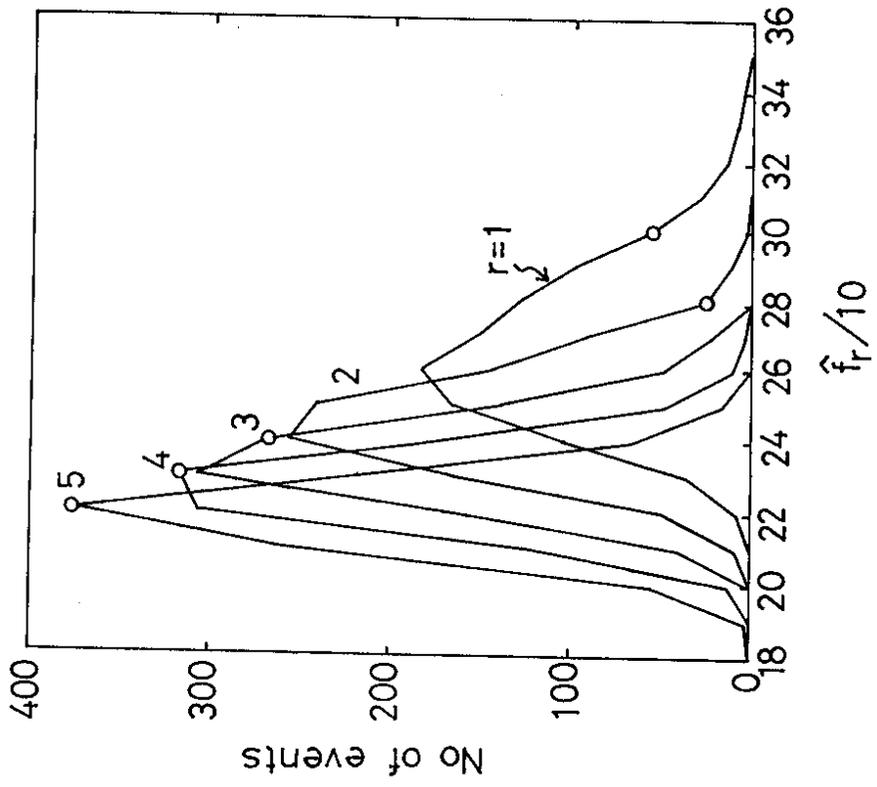


Fig. 8

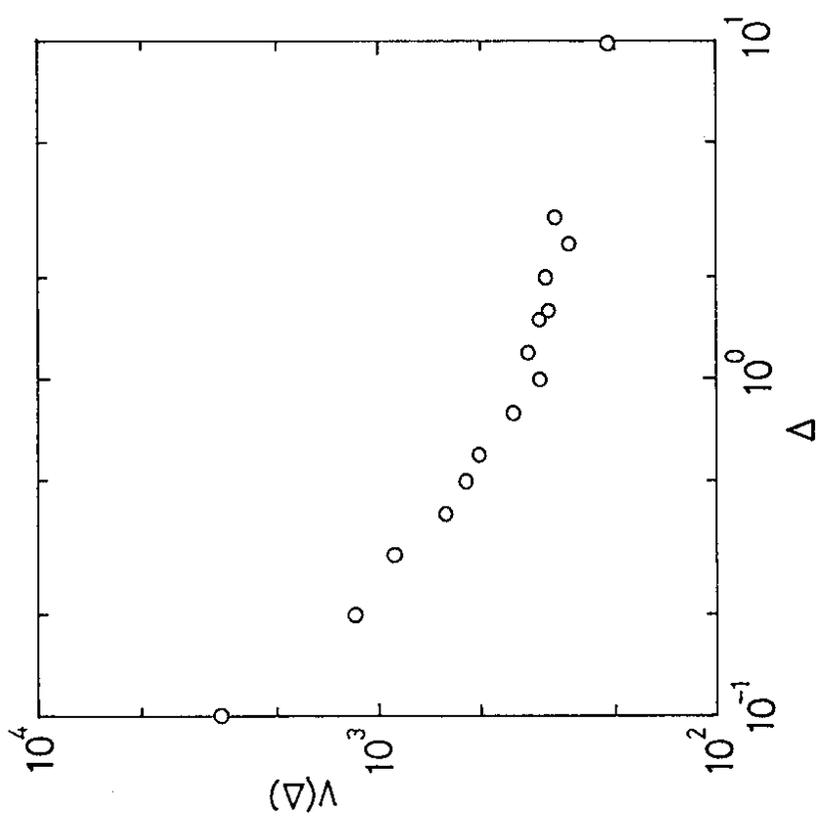


Fig. 7

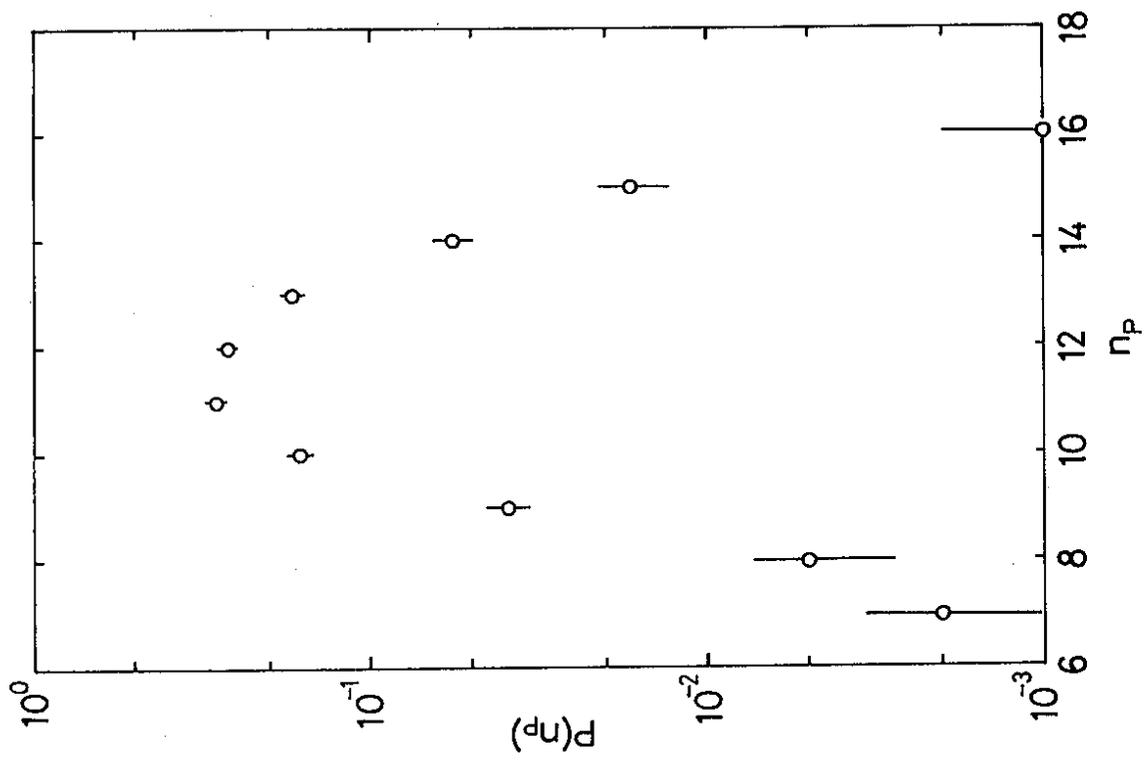


Fig. 12

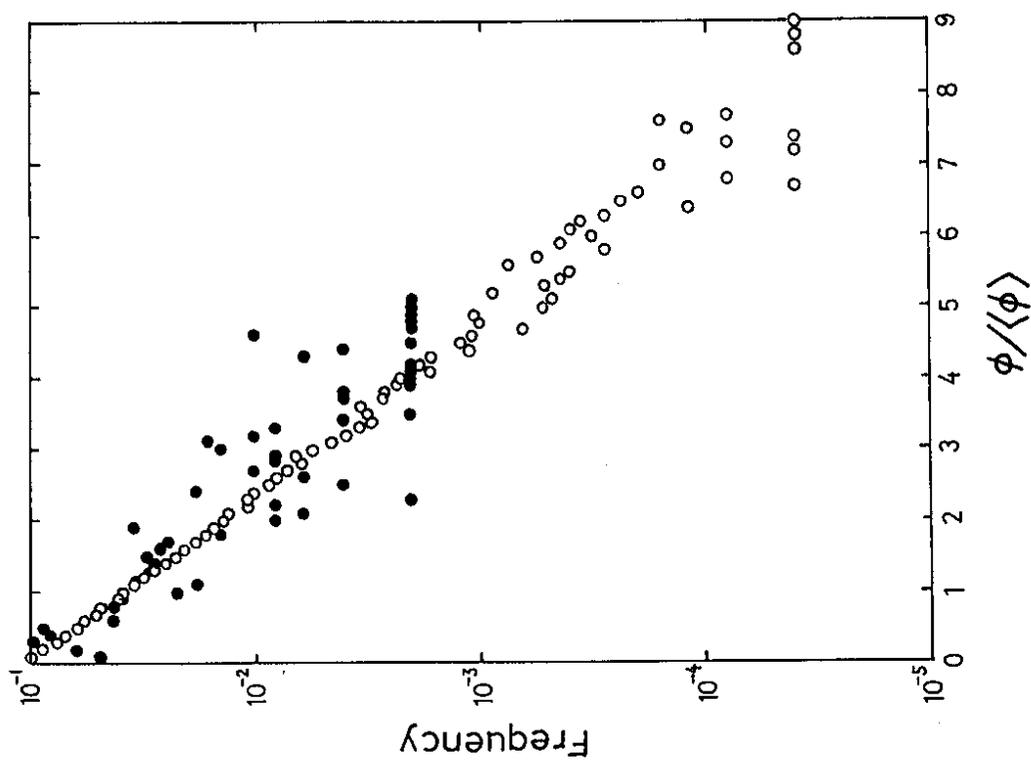


Fig. 11

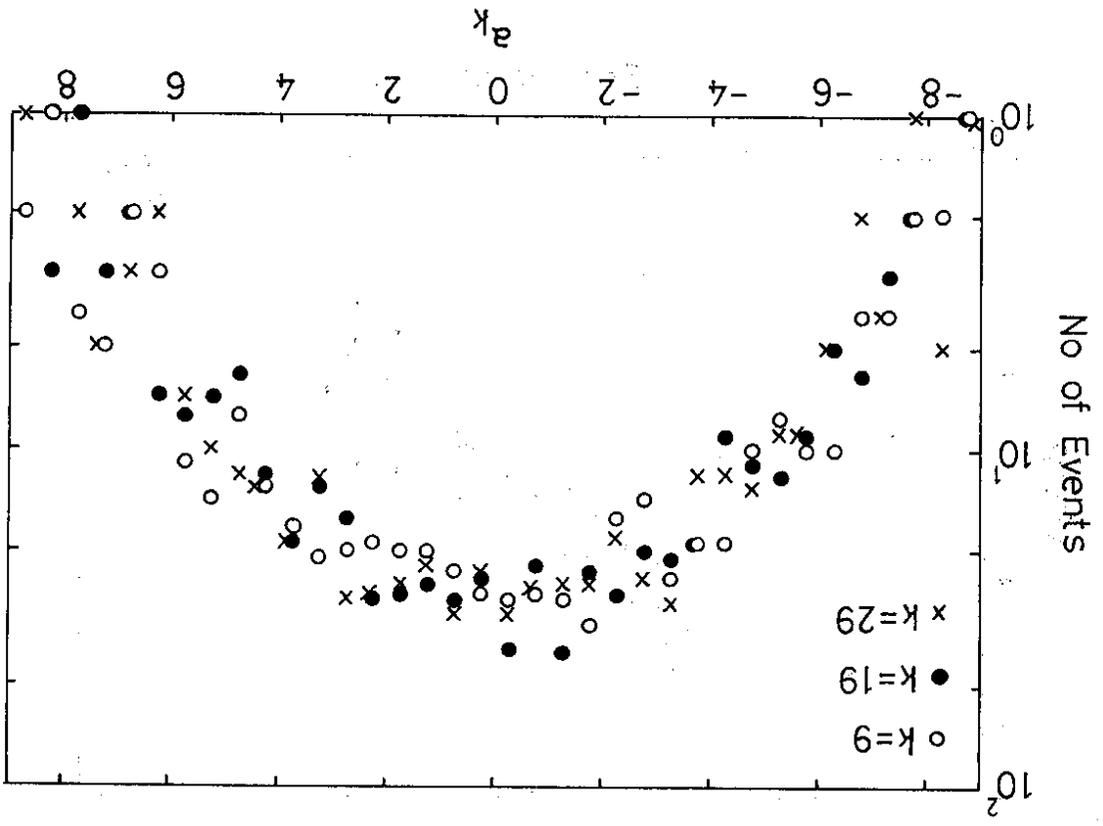


Fig. 14

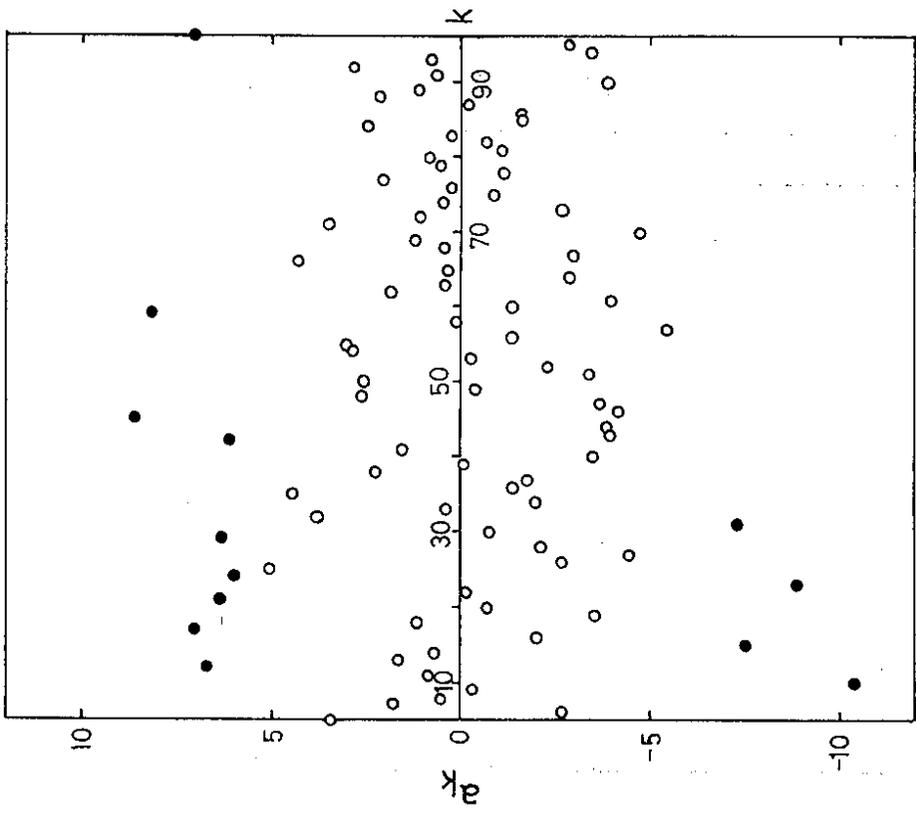


Fig. 13

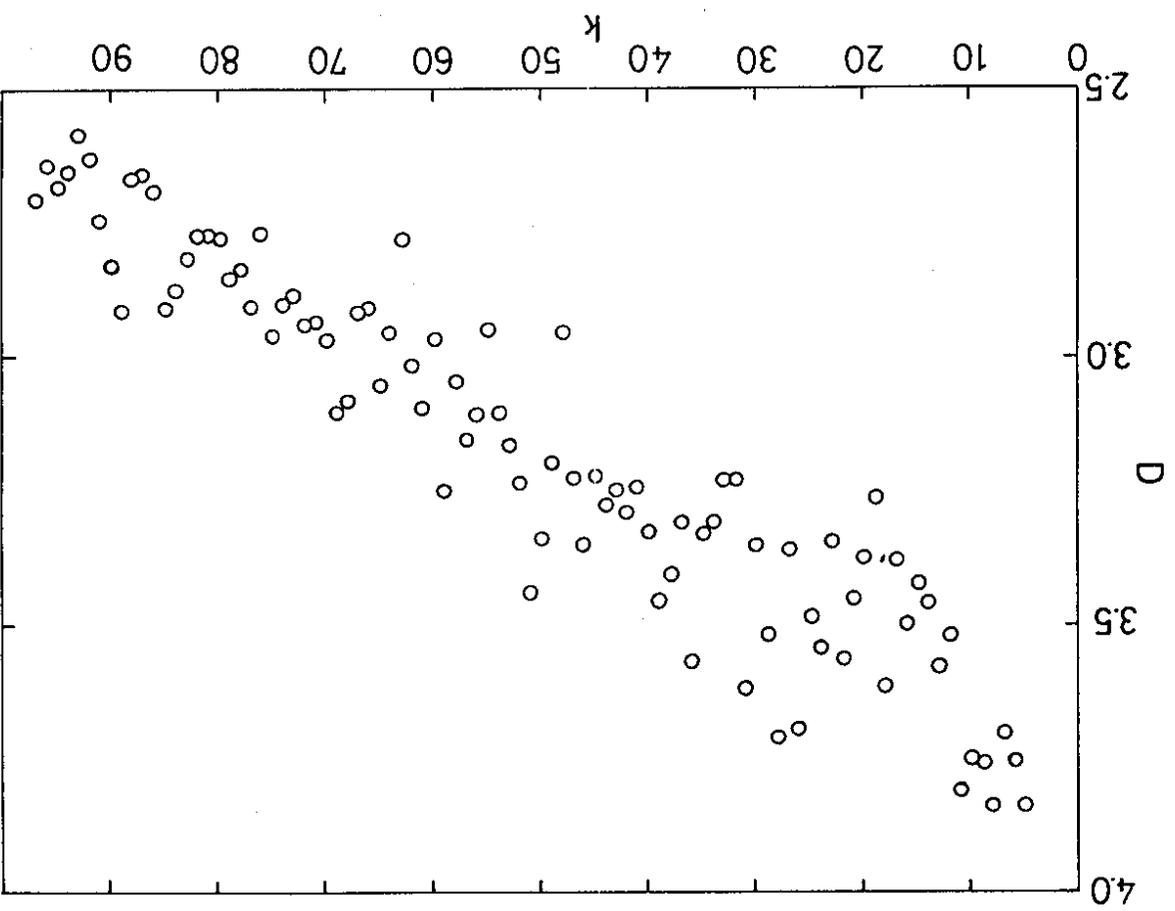


Fig. 15

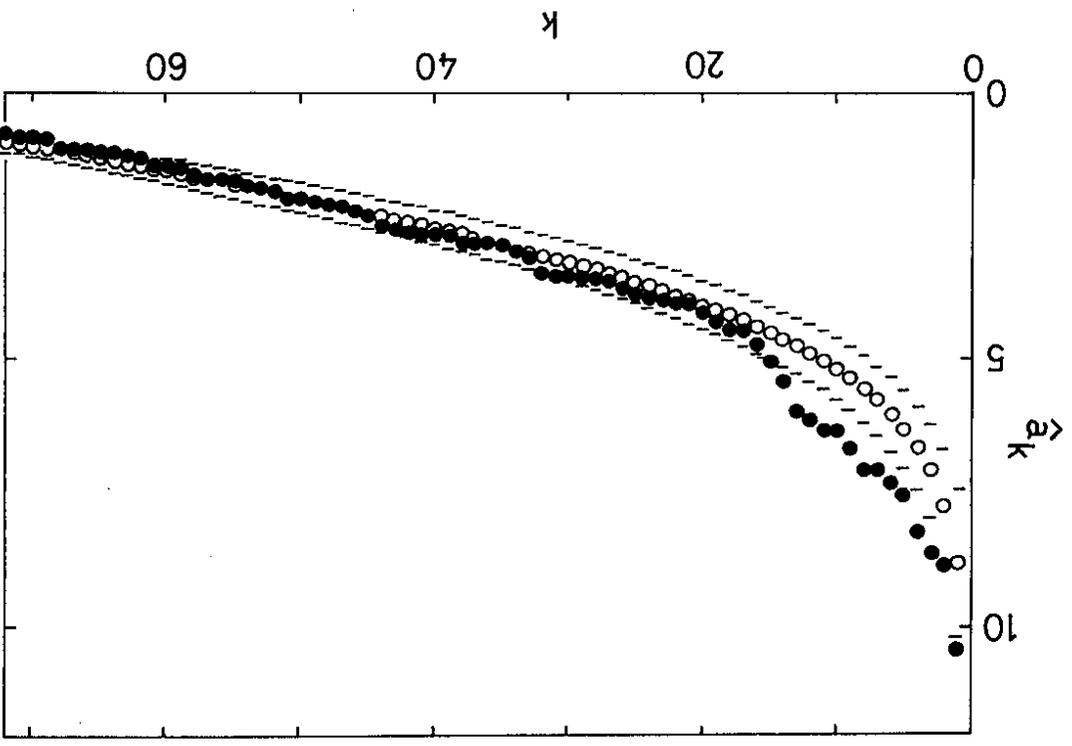


Fig. 16

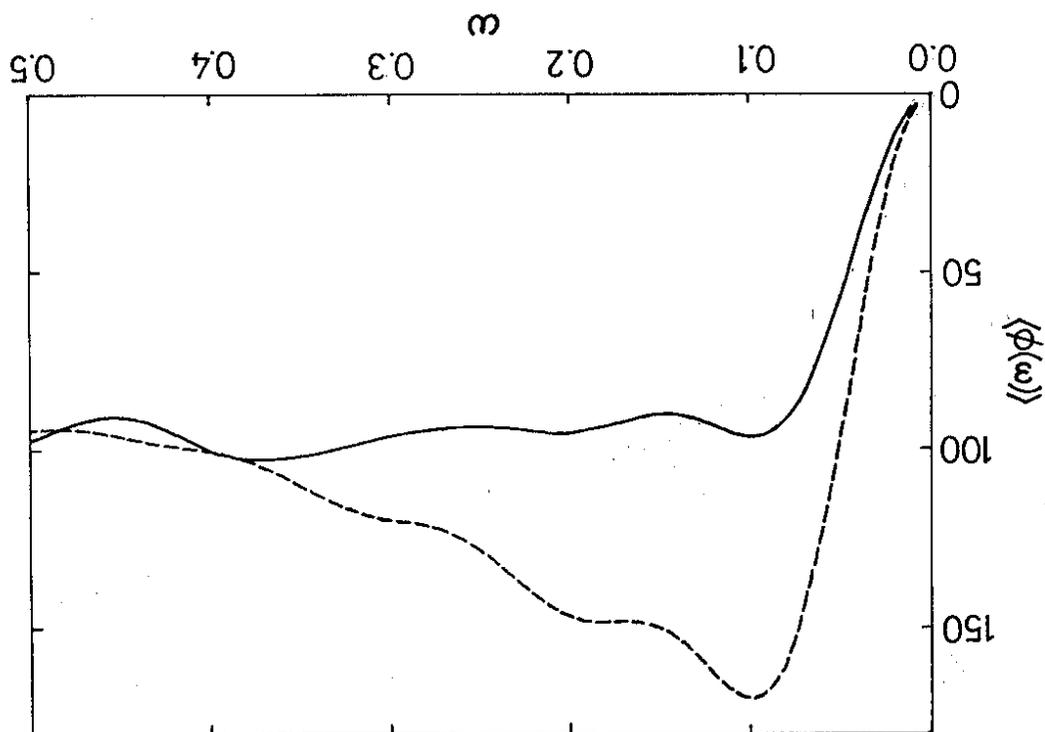


Fig. 17

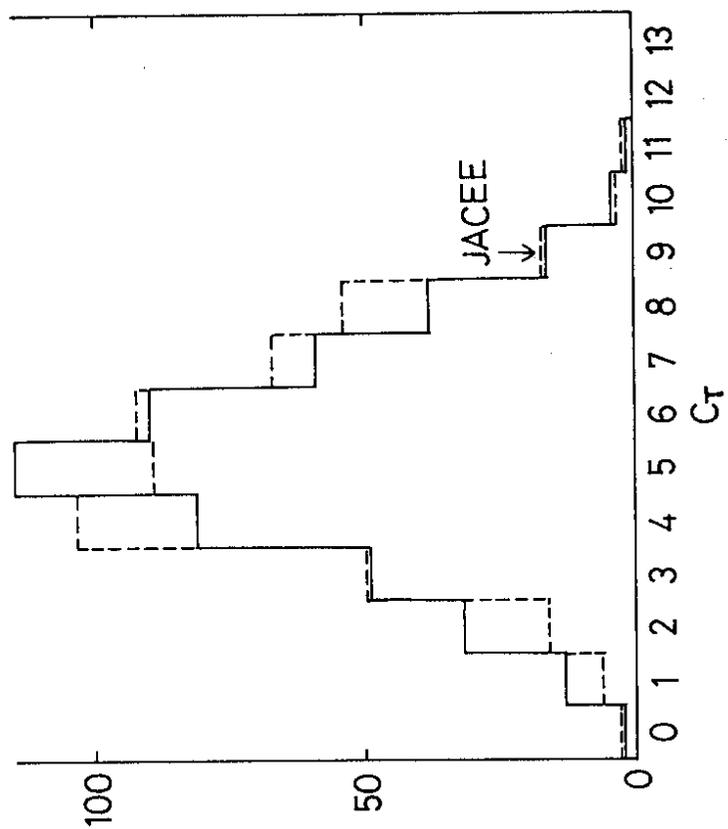


Fig. 18